A Study of a 7-in-a-Row Gomoku Variant with Dual-Symbol Tiles

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Abstract

7-in-a-Row is a two-player abstract strategy game on a 19×19 board.
Each turn a player places a **2×2 tile** that contains **two X's and two O's** in one of six fixed patterns.
A player wins by forming **seven or more** consecutive symbols of their own (horizontal, vertical, or diagonal).
A single placement can complete a line for **both** players at once – a **mutual win**.
We formalise the rules, analyse strategic consequences, and evaluate the choice of k=7.
The game inherits PSPACE-completeness from Gomoku but adds non-zero-sum terminals and rigid tile geometry, making a full solution far beyond current techniques.

1. Introduction

Gomoku is a well-studied connection game in which players alternately place single stones to form five in a row. It is PSPACE-complete on general boards (Reisch, 1980) and solved with a first-player win on 15×15 under perfect play (Allis, 1994). Advances in threat-space search (Allis et al., 1993) and Monte Carlo methods (Kang & Kim, 2016) have deepened understanding of such games.

This work presents 7-in-a-Row, a variant in which moves consist of placing predefined 2×2 tiles containing symbols for both players. This mechanism alters line formation, introduces symmetry in symbol placement, and enables non-zero-sum outcomes. The objectives are to define the game rigorously, characterize its strategic and computational properties, and evaluate the design decision k=7 as the winning threshold.

2. Rules

Component	Description
Board	19×19 grid; cells are empty, **X** (player 1) or **O** (player 2).
Tiles	Six configurations (H1–H2, V1–V2, D1–D2); each contains exactly two X's and two O's.
Move	Choose a tile and any empty 2×2 region; all four cells must be empty.
Win	7 or more consecutive own symbols in a straight line (horizontal, vertical, or either diagonal).
Draw	No legal moves left and neither player has won.
Mutual win	One placement creates more than 7 for **both** players.

Tiles:

X	X
O	O

H1 (XX/OO)

O	O
X	X

H2 (OO/XX)

X	O
X	O

V1 (XO/XO)

O	X
O	X

V2 (OX/OX)

X	O
O	X

D1 (XO/OX)

O	X
X	O

D2 (OX/XO)

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3. Strategic Properties

3.1. Dual-Symbol Placement

Each move increases both players' symbol counts by two. This enforces parity after complete turns and couples offensive and defensive considerations. A placement intended to extend one line may simultaneously enable or block an opponent's path.

3.2. Non-Zero-Sum Outcomes

The mutual win state violates zero-sum assumptions. Standard utility assignments (+1, –1, 0) fail to represent rational play near such terminals. Players may prefer mutual victory over unilateral loss, requiring evaluation functions that incorporate context-dependent payoff vectors.

- **Beyond Zero-Sum:** Traditional two-player deterministic games are zero-sum; one player's win is another's loss. The "win-win" state transforms 7-in-a-Row into a non-zero-sum game, where mutually beneficial outcomes are possible. This introduces a psychological dimension rarely seen in abstract strategy games. A player on the verge of losing might pivot their strategy to force a "win-win," thereby salvaging a draw from a near-certain defeat.

- **Challenges for Artificial Intelligence:** This dynamic poses a significant challenge for AI development. A standard AI with a utility function that only maximizes its own score (e.g., +1 for a win, -1 for a loss, 0 for a draw) would likely never initiate a "win-win." To create a human-like AI for 7-in-a-Row, one would need a more nuanced utility function capable of valuing the "win-win" state based on context. Investigating the performance of MCTS (Kang & Kim, 2016) with such modified utility functions is a promising research direction.

- **Human-Computer Interaction:** The non-zero-sum nature also opens avenues for studying human player strategies, particularly how different player pairings (competitive vs. cooperative) approach the "win-win" condition.

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4. Analysis of the Winning Condition (k=7)

The choice of **seven** as the winning length is a pivotal design decision. This section critically examines its suitability, drawing on principles from generalized *m,n,k*-connection games and the unique constraints of dual-symbol tile placement.

4.1. Comparison with Gomoku

In standard Gomoku, k=5 yields a first-player advantage on large boards. Increasing k strengthens defense. In 7-in-a-Row, tile rigidity constrains line construction more than free placement, making k=7 functionally longer than in Gomoku despite the numerical value.

4.2. Balance Implications

- **k=6**: Likely favors first player due to rapid threat development via horizontal/vertical tiles.
- **k=7**: Intended neutral zone; dual symbols support blocking and mutual completion.
- **k=8 or 9**: High draw probability from board fragmentation before line completion.

4.3. Board Size Interaction

A 19×19 grid supports up to 90 tile placements. The maximum diagonal is 37 cells, so k=7 is well below saturation. Fragmentation—regions smaller than 2×2—reduces effective space over time, amplifying the defensive bias of larger k.

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5. Computational Complexity

- **Initial branching factor** – 6×18^2 = **1,944** (vs. 361 in Gomoku).
- **State space** – bounded above by 3^{361} but reduced by block constraints.
- **Self-pruning** – isolated cells and narrow corridors eliminate moves late-game.
- **Solvability** – strategy-stealing does not apply; non-zero-sum terminals prevent standard proofs.

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6. Conclusion

7-in-a-Row enriches Gomoku with rigid 2×2 tiles and a genuine win-win possibility.
The k=7 threshold appears balanced, yet its interaction with tile geometry and fragmentation merits further empirical study.
The game resists standard analytical methods — including minimax, threat-space search, and strategy-stealing — due to its asymmetric move set, high branching factor, and non-zero-sum terminals. **No proof of optimal play is known, and the structure suggests that achieving one will require fundamentally new ideas in AI or game theory.**
Future work may target AI that reasons about mutual wins, or systematic balance tests across winning lengths from 5 to 10.

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7. References

[1] Reisch, S. (1980). *Gobang is PSPACE-complete*. Acta Informatica, 13, 59–66.

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[4] Kang, J. H., & Kim, H. J. (2016). *Effective Monte-Carlo tree Search Strategies for Gomoku AI*. International Journal of Circuit Theory and Applications, 10, 4841–4843.

[5] Wikipedia: Gomoku. https://en.wikipedia.org/wiki/Gomoku

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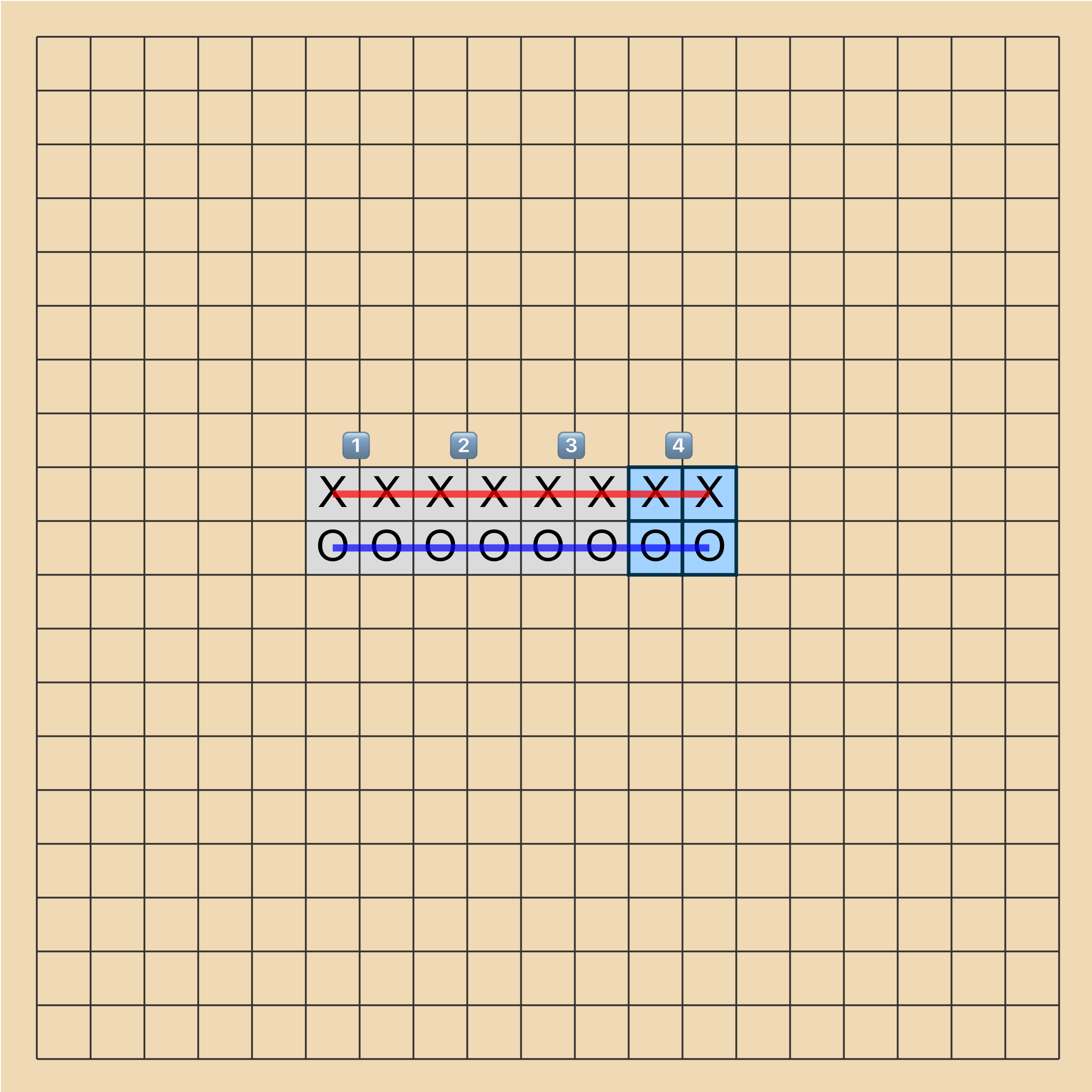
Appendix A: Sample Game Scenarios

Example Win-Win Scenario

The following board state illustrates a "win-win" scenario. Player X has a horizontal threat, and Player O has a vertical threat. The placement of the final tile (highlighted in blue) simultaneously completes a 7-in-a-row for both players.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